Analog Electronic

ENEE236

BJT AC Analysis

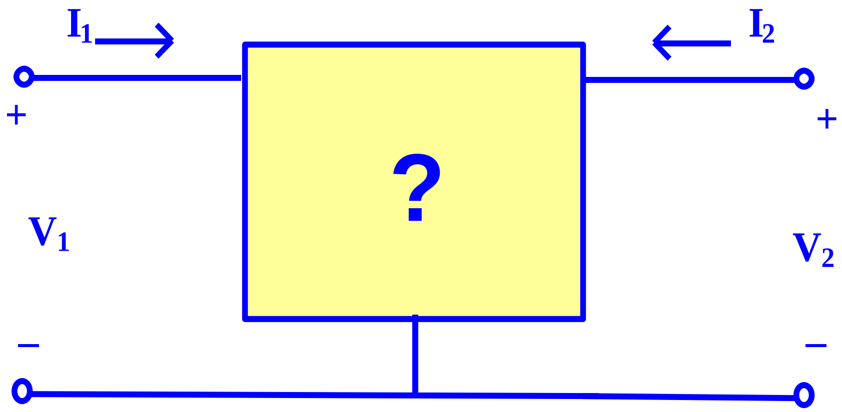
Chapter 5

Small Signal ac Equivalent Circuit

- In order to simplify the analysis, we replace the Transistor by an equivalent circuit (model)
- An AC model represents the AC characteristics of the transistor.
- A model uses circuit elements that approximate the behavior of the transistor.
- There are two models commonly used in small signal AC analysis of a transistor:
    - **r_e model**
    - **Hybrid equivalent model**

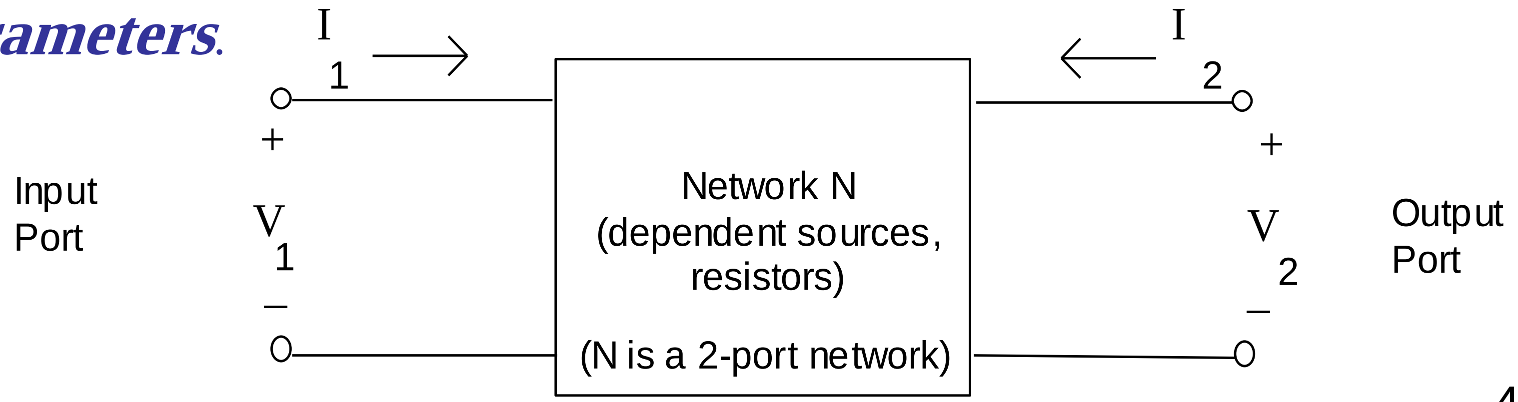
Modeling Two-Port Networks

- Two-port parameters can be determined for a given network.
- Additionally, two-port parameters might be specified for a certain device by the manufacturer (such as h-parameter values for a transistor).
- How are these parameters used?
- They are used to form a circuit model for the device or circuit. A circuit model is developed using the two-port parameter equations.

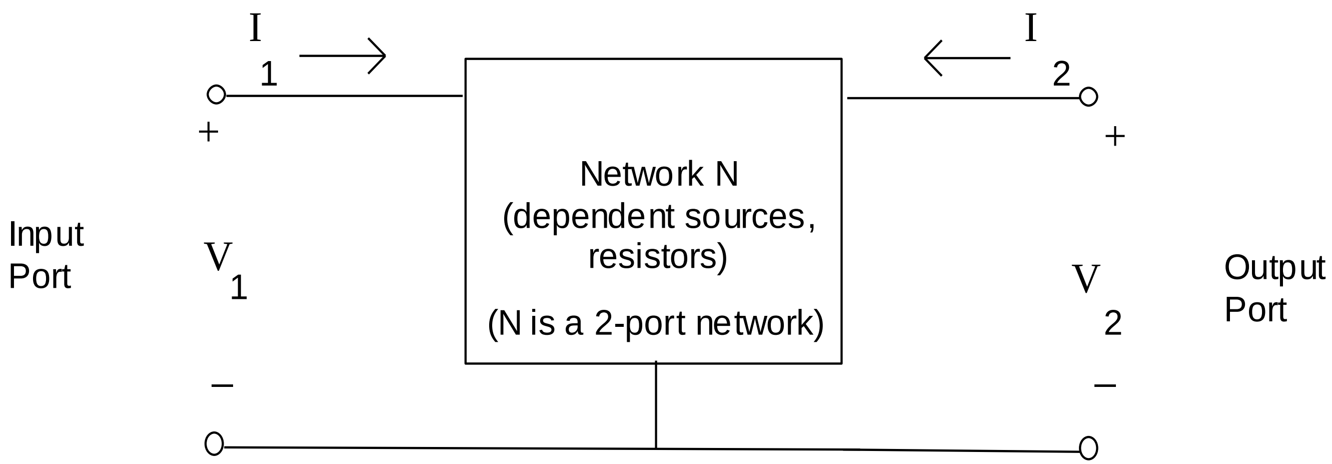


Two-port networks

- Suppose that a network N has two ports as shown below. How could it be represented or modeled?
- A common way to represent such a network is to use one of 6 possible ***two-port networks***.
- These networks are circuits that are based on one of 6 possible sets of ***two-port equations***. These equations are simply different combinations of two equations that relate the variables V_1 , V_2 , I_1 , and I_2 to one another. The coefficients in these equations are referred to as ***two-port parameters***.



Note that I_1 , I_2 , V_1 , and V_2 are labeled as shown by convention. Often there is a common negative terminal between the input and the output so the figure above could be redrawn as:



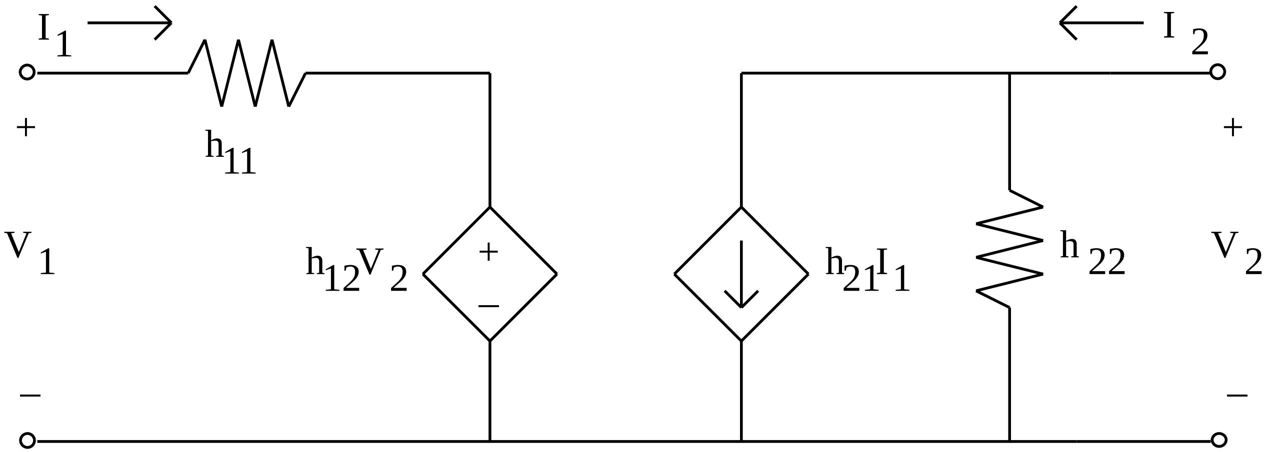
Development of the h-parameter model:

One possible circuit model could be developed by treating one of the two-port parameter equations as a KVL equation and the other as a KCL equation (illustrate). This results in the following circuit.

h - parameter equations :

$$V_1 = h_{11} \cdot I_1 + h_{12} \cdot V_2$$

$$I_2 = h_{21} \cdot I_1 + h_{22} \cdot V_2$$



$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2 = 0}$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1 = 0}$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2 = 0}$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1 = 0}$$

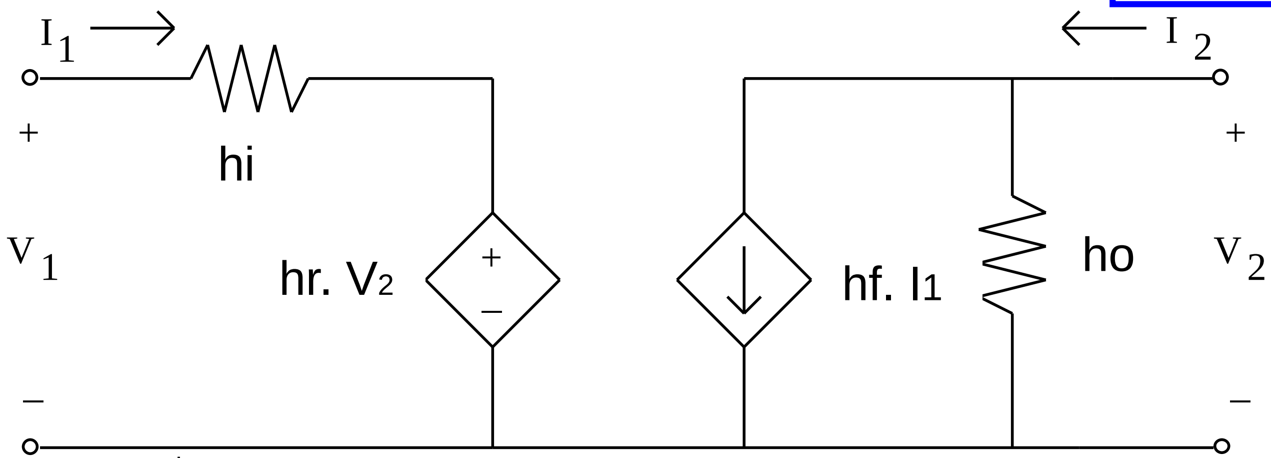
Development of the h-parameter model of BJT:

For A BJT the equivalent h parameter model can be described by the following equations:

h - parameter equations:

$$V_1 = h_i \cdot I_1 + h_r \cdot V_2$$

$$I_2 = h_f \cdot I_1 + h_o \cdot V_2$$

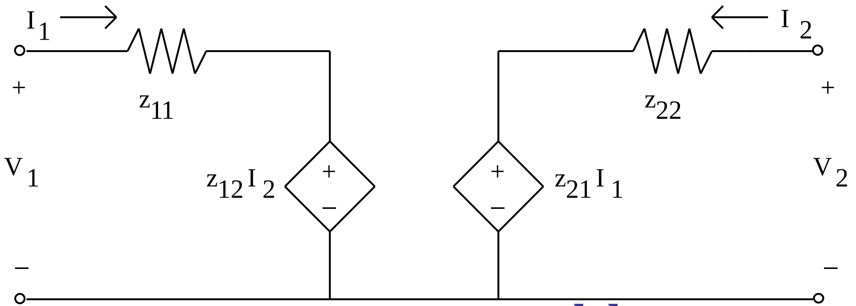


$$h_i = \left.\frac{V_1}{I_1}\right|_{V_2=0} \qquad h_r = \left.\frac{V_1}{V_2}\right|_{I_1=0}$$

$$h_f = \left.\frac{I_2}{I_1}\right|_{V_2=0} \qquad h_o = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

Summary:

Note: This page is for information only

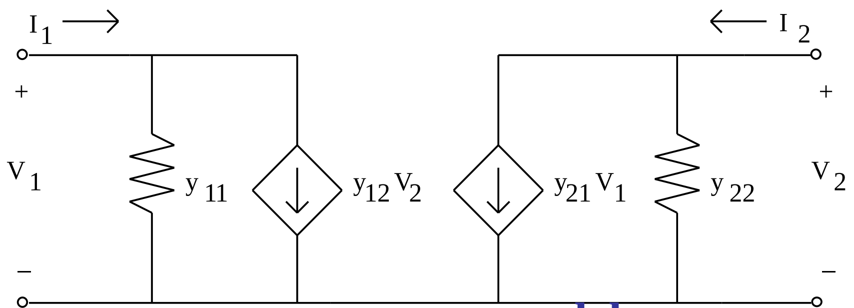


z-parameter model

z-parameter equations:

$$V_1 = z_{11} \cdot I_1 + z_{12} \cdot I_2$$

$$V_2 = z_{21} \cdot I_1 + z_{22} \cdot I_2$$

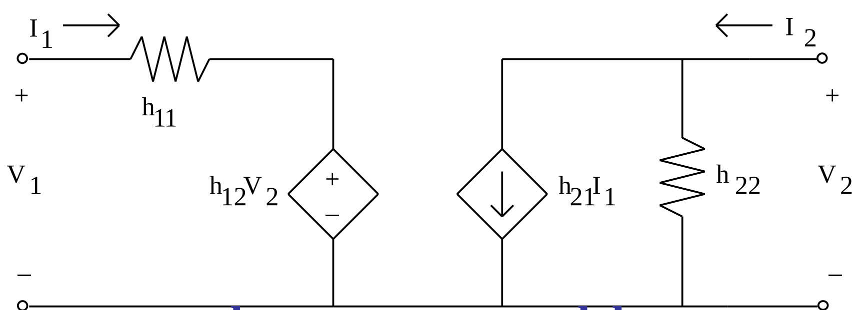


y-parameter model

y-parameter equations:

$$I_1 = y_{11} \cdot V_1 + y_{12} \cdot V_2$$

$$I_2 = y_{21} \cdot V_1 + y_{22} \cdot V_2$$



h-parameter model

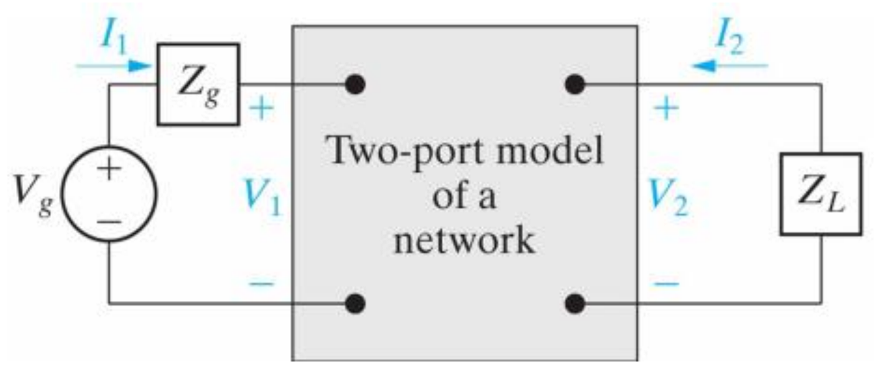
h-parameter equations:

$$V_1 = h_{11} \cdot I_1 + h_{12} \cdot V_2$$

$$I_2 = h_{21} \cdot I_1 + h_{22} \cdot V_2$$

BJT Configurations

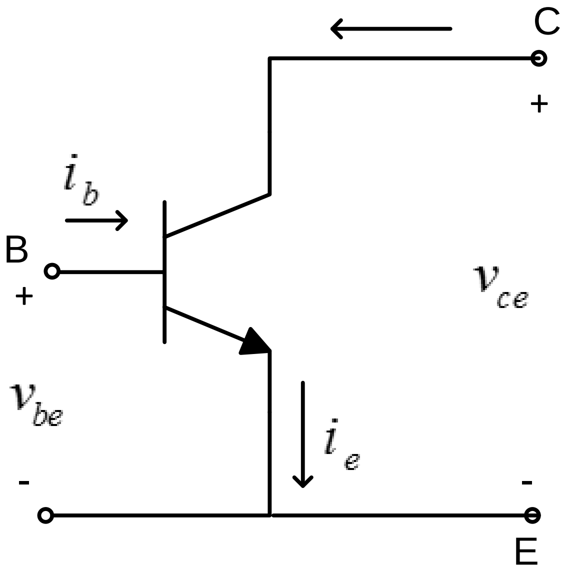
- Common Emitter
- Common Base
- Common Collector



Terminated Two port network
Includes source and load

Common Emitter Configuration

(inverting configuration, provides voltage and current gain)

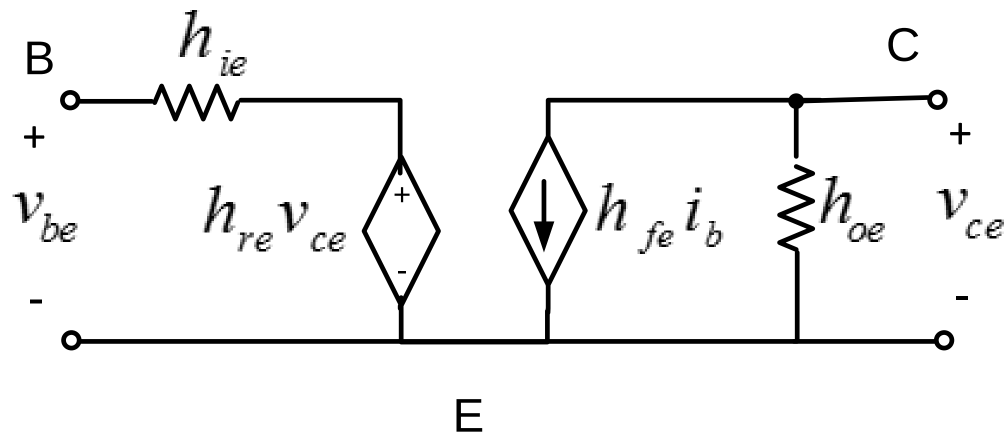


h - parameter equations:

$$V_{be} = h_{ie} \cdot I_b + h_{re} \cdot V_{ce}$$

$$I_c = h_{fe} \cdot I_b + h_{oe} \cdot V_{ce}$$

Detailed Model



Typical Data sheet parameter values

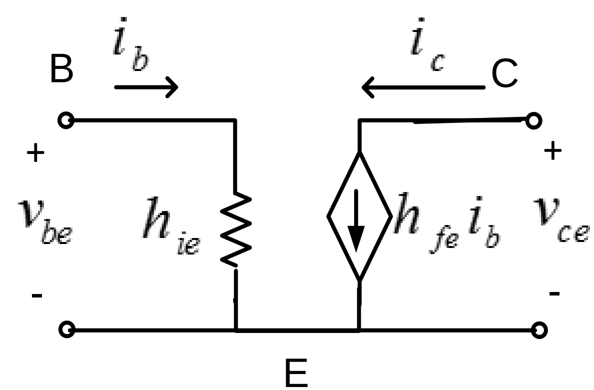
$h_{ie} \approx 1600\,\Omega$

$h_{re} \approx 0.0002$

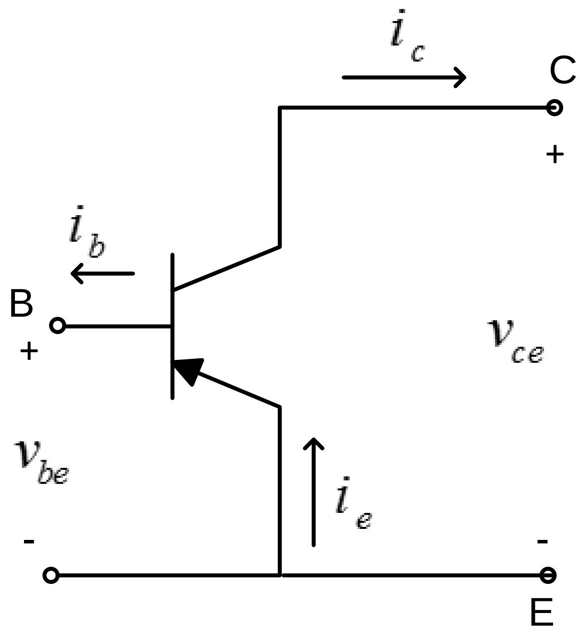
$h_{fe} \approx 80$

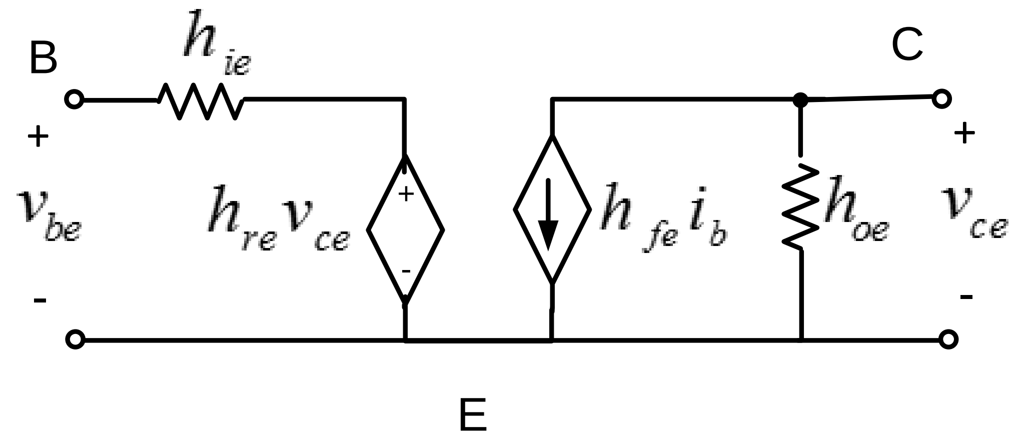
$h_{oe} \approx 20.10^{-6}$ Siemens

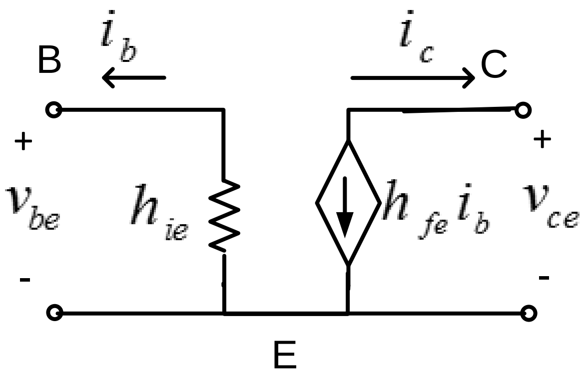
Simplified Model



Common Emitter and Common Collector Configuration







Value of hie

Base Emitter is a pn junction similar to a diode
hie is the dynamic resistance of the pn junction

In a diode:

$$r_d = \frac{V_T}{I_{DQ}} \Rightarrow$$

$$h_{ie} = \frac{V_T}{I_{BQ}} = \frac{V_T}{\frac{I_{CQ}}{h_{fe}}} = \frac{h_{fe} V_T}{I_{CQ}}$$

I_{BQ} dc value of base current

I_{CQ} dc value of collector current

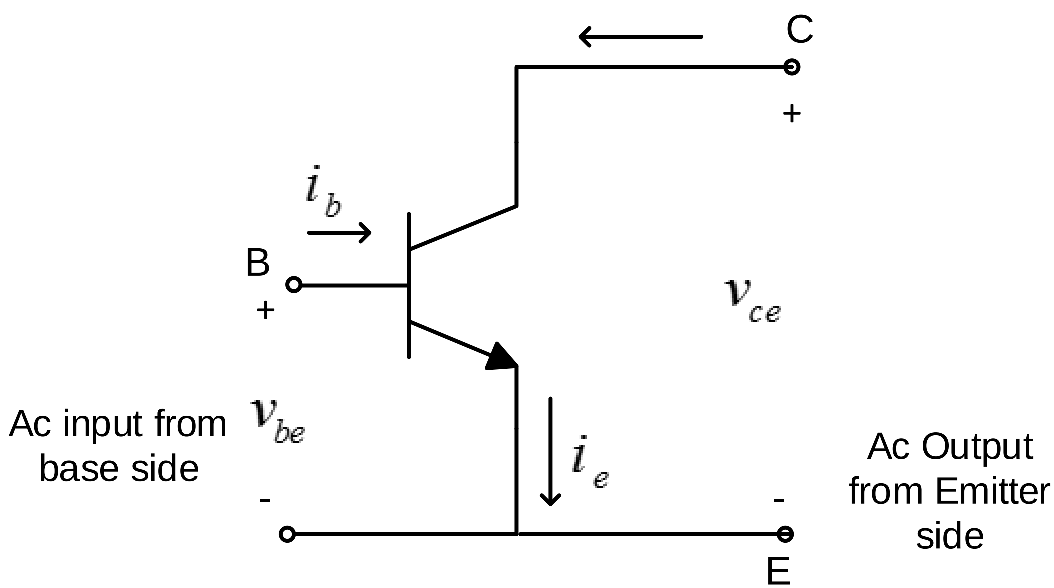
$$h_{fe} = \beta$$

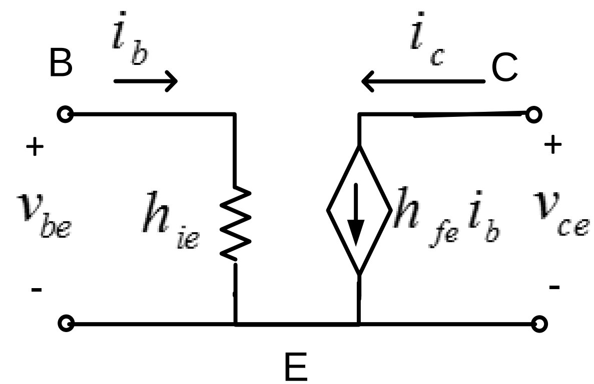
$$V_T = 25.69 \text{ mV} \ @ \ 25\ ^oC$$

Common Collector

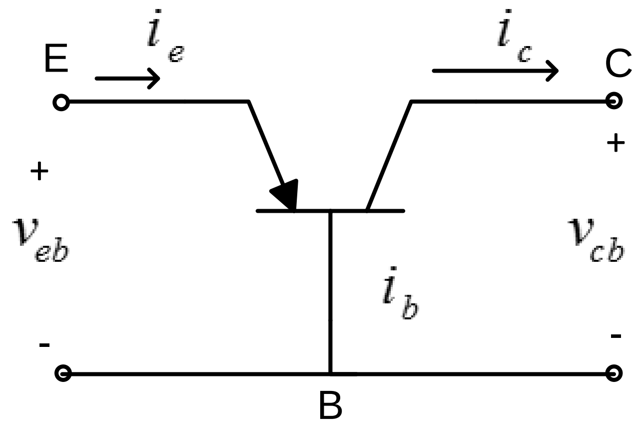
provides current gain and no voltage gain)

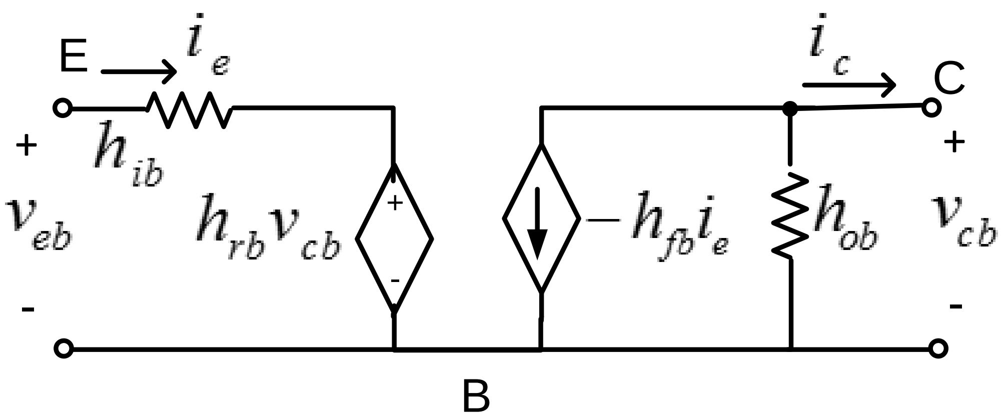
Same Model of Common Emitter will be used due to the similarities between them and for simplicity





Common-Base Configuration





h - parameter equations:

$$V_{eb} = h_{ib} \cdot I_e + h_{rb} \cdot V_{cb}$$

$$I_c = h_{fb} \cdot I_e + h_{ob} \cdot V_{cb}$$

$$h_{ib} = \left.\frac{V_{EB}}{I_E}\right|_{V_{CB}=0}$$

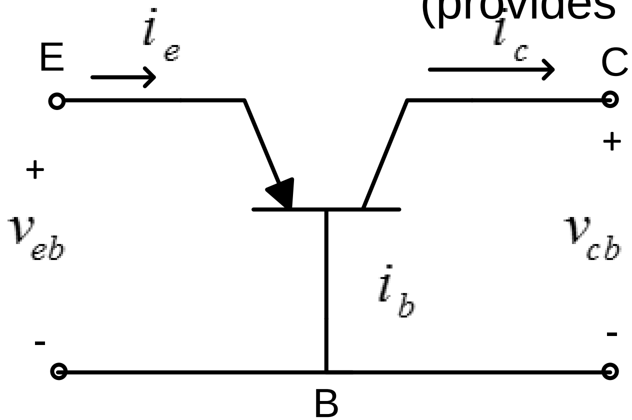
$$h_{rb} = \left.\frac{V_{EB}}{V_{CB}}\right|_{I_E=0}$$

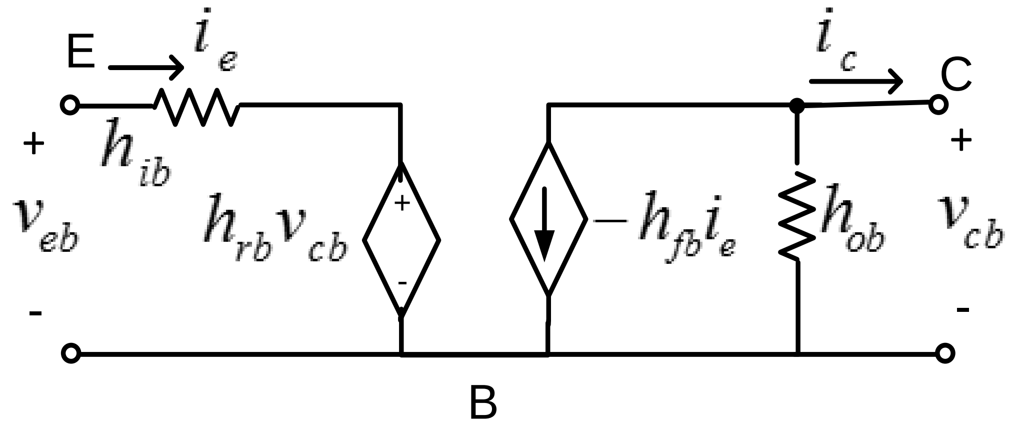
$$h_{fb} = \alpha = \left.\frac{I_C}{I_E}\right|_{V_{CB}=0}$$

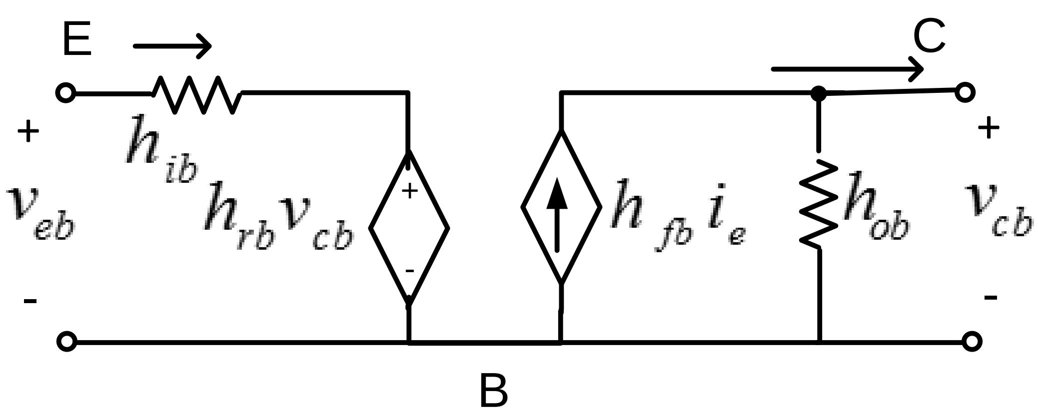
$$h_{ob} = \left.\frac{I_C}{V_{CB}}\right|_{I_E=0}$$

Common-Base Configuration

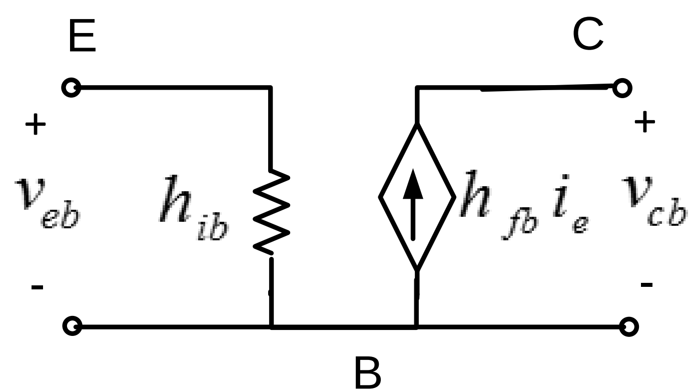
(provides current gain and some voltage gain)







Simplified Equivalent Circuit



Common-Base Configuration

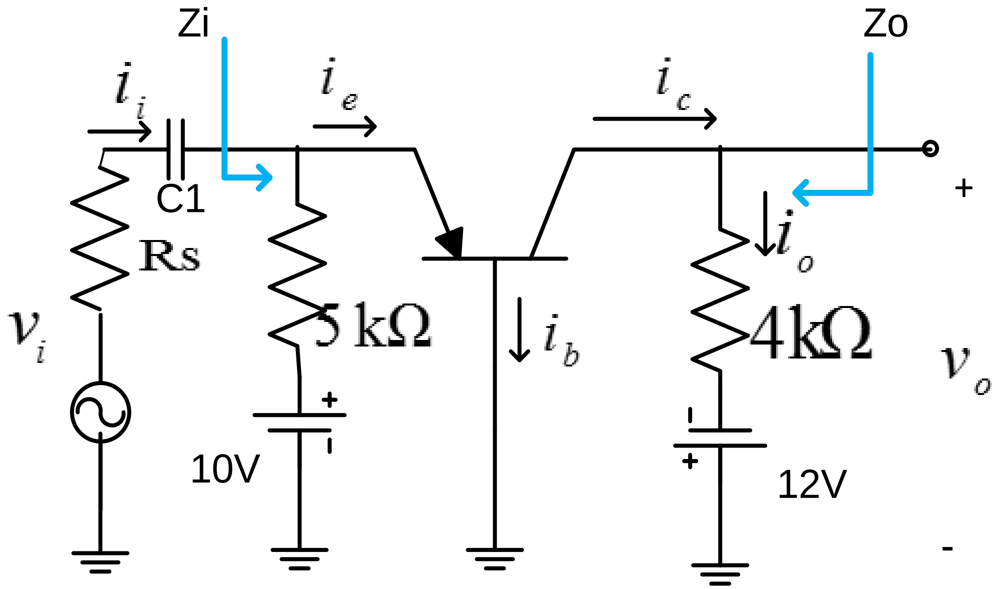
$$h_{ib} = \frac{V_T}{I_{EQ}}$$

$$h_{fb} = \alpha$$

$$V_T = 25.69 \ \text{mV} \ \ @ \ 25\ ^{o}\text{C}$$

$$h_{ie} > h_{ib}$$

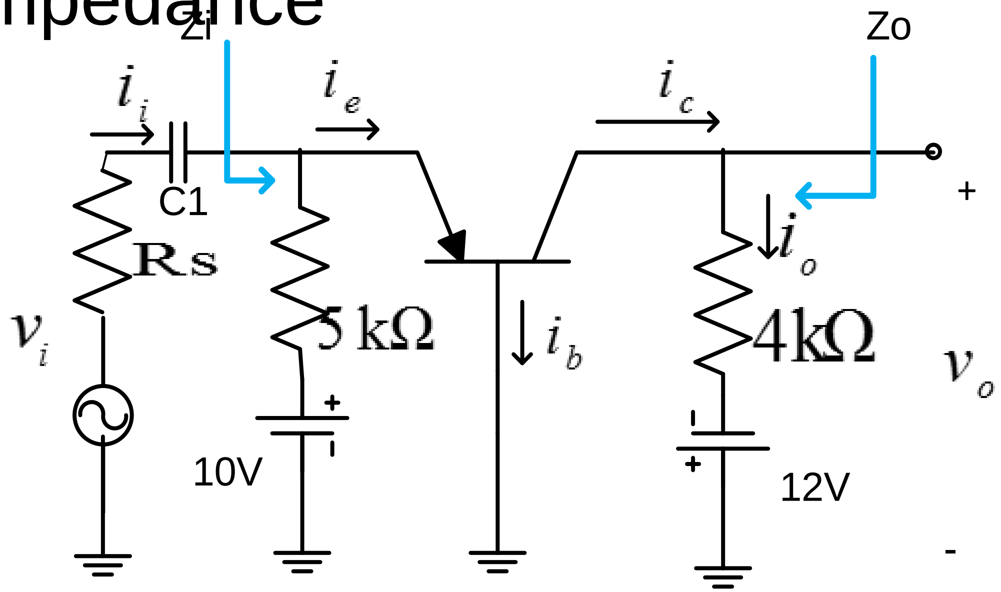
BJT Amplifier Analysis Example



BJT Amplifier Analysis

When Analyzing Amplifier Circuits, we usually want to find some or all of the following quantities:

1) Av=Vo/Vi, small signal voltage gain
2) Ai=io/ii,   small signal current gain
3) Zi   Input Impedance
4) Zo   Output Impedance



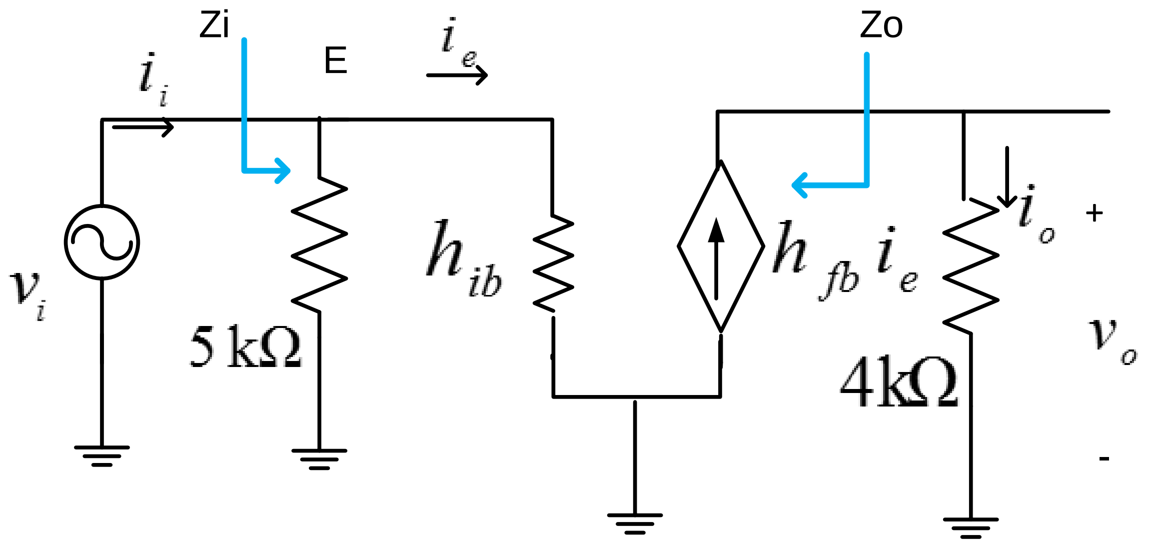
BJT Amplifier Analysis

Solution: (with Rs=0)

We draw the ac small signal equivalent circuit

Capacitors ==> replaced by short circuit

DC sources are killed ,



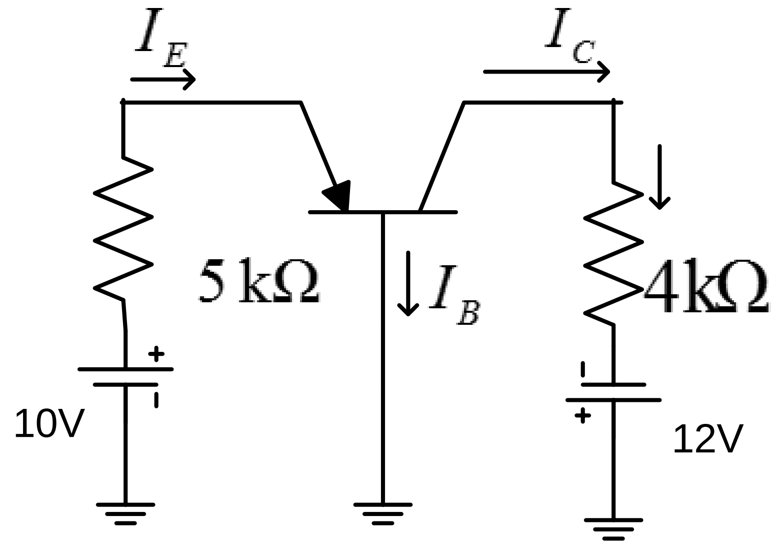
$$h_{ib} = \frac{V_T}{I_{EQ}}$$

$$h_{fb} = \alpha \cong 1$$

I_{EQ} must be calculated from DC analysis

DC Analysis

DC Equivalent Circuit:
-Cap ==> open
-Kill ac sources ==>



$$10 = 5\,k\Omega\,.I_{EQ} + V_{EB}$$

$$I_{EQ} = \frac{10 - 0.7}{5\,k\Omega} = 1.86\,mA$$

$$h_{ib} = \frac{V_T}{I_{EQ}} = \frac{25.69\,mV}{1.86\,mA} = 13.98\,\Omega$$

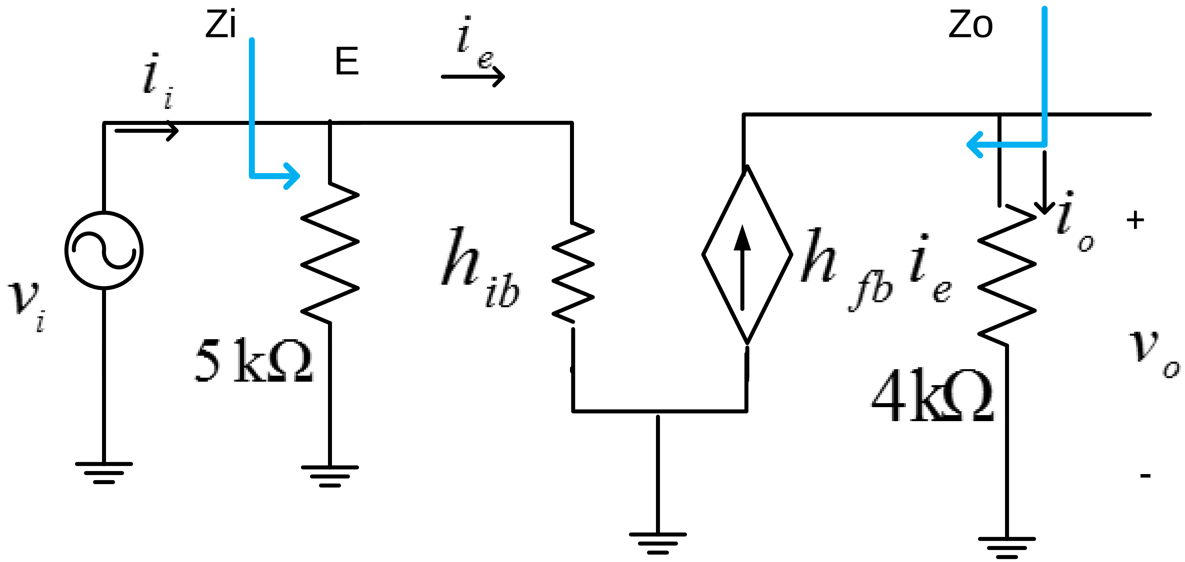
Ac ss equivalent circuit

$$1)\ A_V = \frac{v_o}{v_i}$$

$$v_o = i_o . 4\,k\Omega$$

$$i_o = h_{fb} . i_e$$

$$i_e = \frac{v_i}{h_{ib}}$$



$$A_V = \frac{v_o}{v_i} = \frac{v_o}{i_o} . \frac{i_o}{i_e} . \frac{i_e}{v_i}$$

$$\Rightarrow$$

$$A_V = (4\,k\Omega).(h_{fb}).\left(\frac{1}{h_{ib}}\right)$$

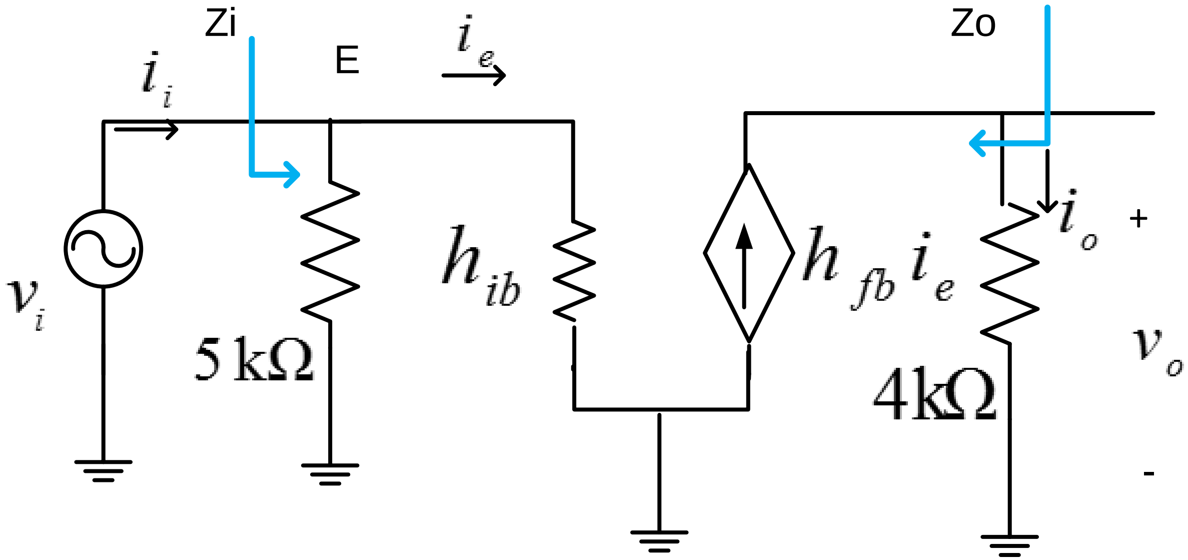
$$= (4\,k\Omega).(1).\left(\frac{1}{13.98}\right) = 286 > 1$$

Current Gain Ai

$$2)\ A_i = \frac{i_o}{i_i}$$

$$i_o = h_{fb} . i_e$$

$$i_e = i_i \frac{5\,k\Omega}{5\,k\Omega + h_{ib}}$$

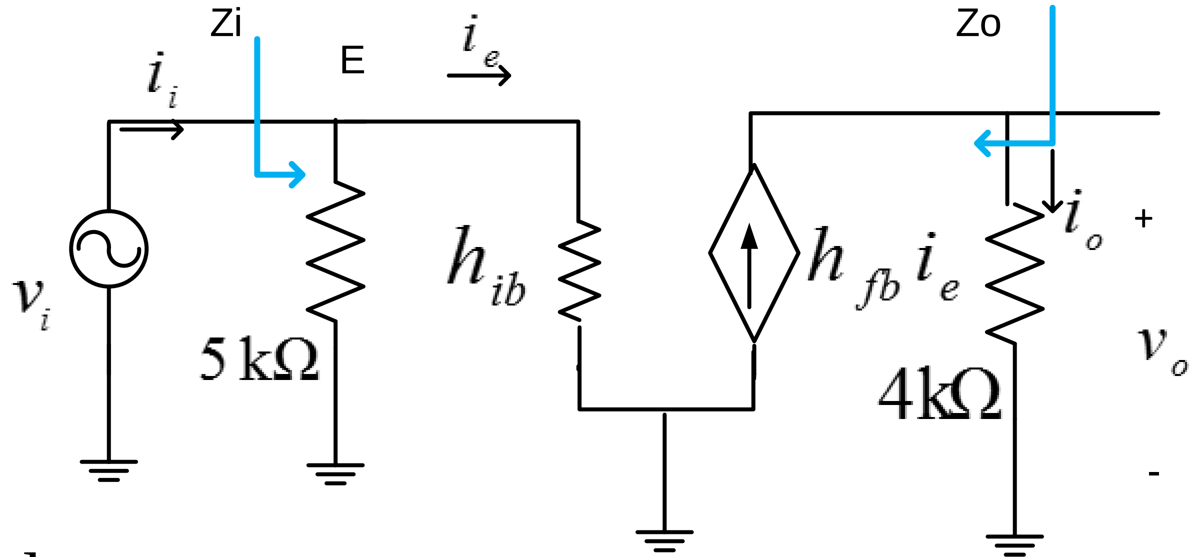


$$\Rightarrow A_i = \frac{i_o}{i_i} = \frac{i_o}{i_e} . \frac{i_e}{i_i}$$

$$\Rightarrow A_i = (h_{fb})\left(\frac{5\,k\Omega}{5\,k\Omega + h_{ib}}\right)$$

$$= (1)\left(\frac{5\,k\Omega}{5\,k\Omega + 13.98}\right) < 1$$

Zi & Zo



3) Input Impedance

$$Z_{\mathrm{i}} = \left(h_{ib} \,//\, 5\,\mathrm{k\Omega}\right) = \left(\frac{h_{ib}.5\,\mathrm{k\Omega}}{5\,\mathrm{k\Omega} + h_{ib}}\right)$$

4) Output Impedance

$$Z_o\Big|_{\text{all independant sources killed (i.e. Vi=0 or short)}} = 4\,\mathrm{k\Omega}$$

With Presence of Rs

with R_S

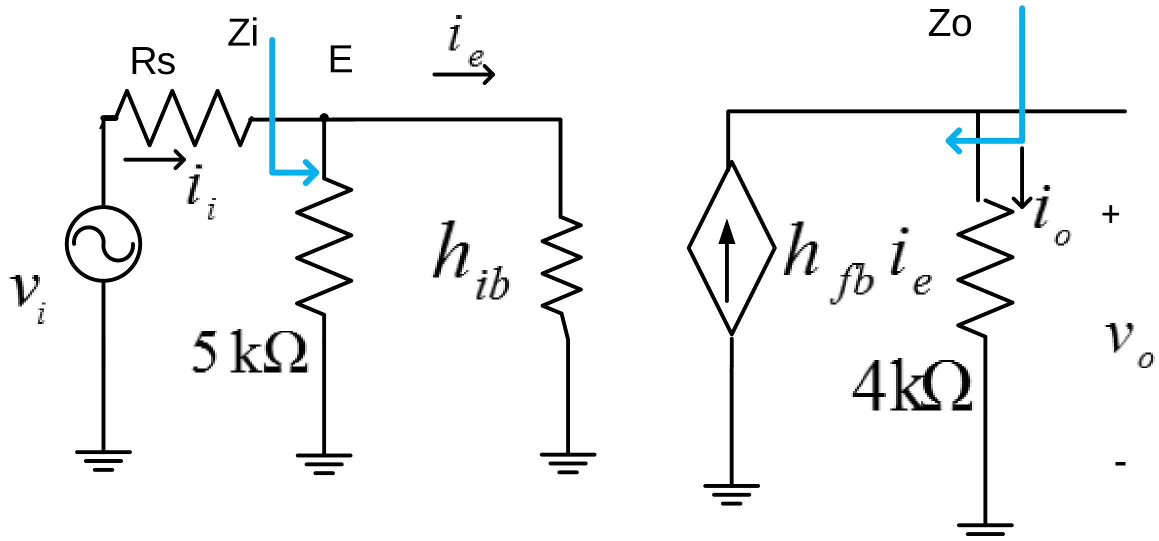
$$i_i = \frac{v_i}{Z_i + R_s}$$

$\text{For Rs} = 50\ \Omega$

$A_V = 62.5$

$\text{For Rs} = 10\ \text{k}\Omega$

$A_V = 0.4$

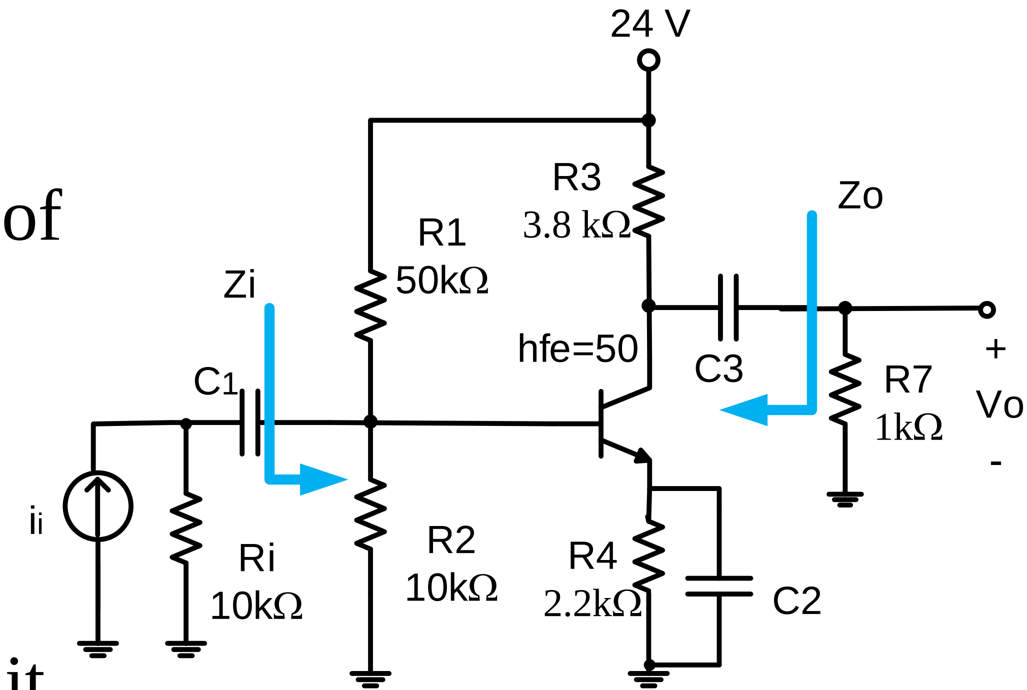


Example: Common Emitter (CE)

1) From DC Analysis,

we find Q - point and value of

$$h_{ie} = \frac{V_T}{I_{BQ}}$$

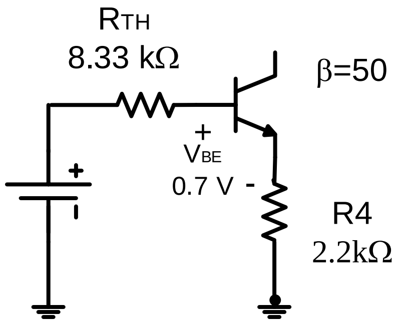


Thevenin's equivalent circuit

as seen from the base

$$V_{TH} = \frac{10\ k\Omega}{10\ k\Omega + 50\ k\Omega} .24\ V = 4\ V$$

$$R_{TH} = 10\ k\Omega\Omega // 5k\Omega = 8.33\ k\Omega$$

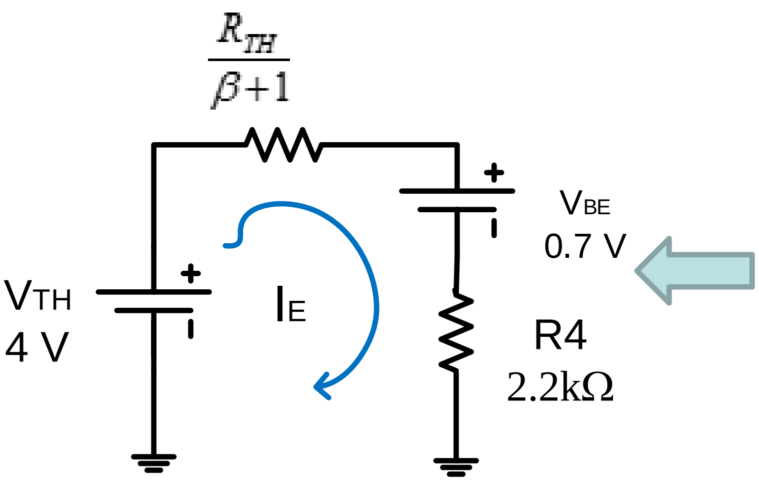


$$4 = 8.33\,\mathrm{k\Omega}\,.\,I_B + V_{BE} + 2.2\,\mathrm{k\Omega}.\,I_E$$

But, $I_E = (1+\beta)I_B$

$$\text{Solve for } I_E = \frac{4-0.7}{\frac{8.33\,\mathrm{k\Omega}}{(1+50)} + 2.2\,\mathrm{k\Omega}} = 1.4\,\mathrm{mA}$$

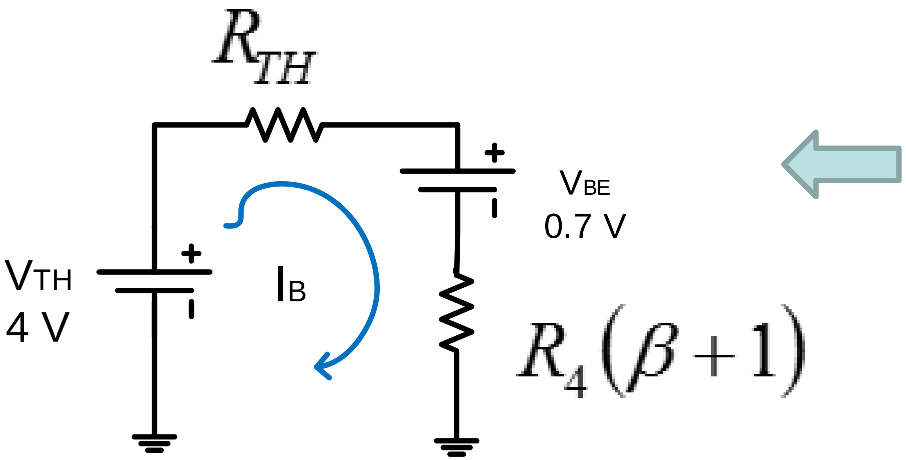
$$h_{ie} = \frac{V_T}{I_{BQ}} = \frac{25.69\,\mathrm{mV}}{\frac{1.4\,\mathrm{mA}}{51}} = 928\,\Omega$$



Here we have base reflected to emitter

$$I_B \Rightarrow I_E = (\beta+1)I_B$$

$$R_B \Rightarrow \frac{R_B}{\beta+1}$$

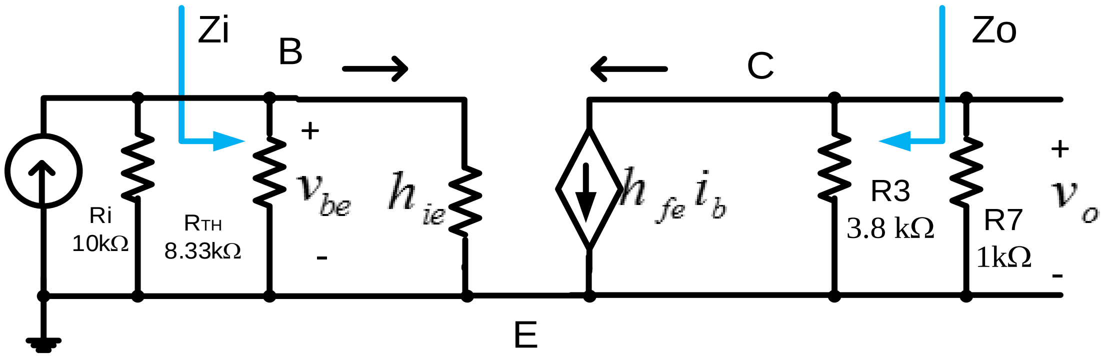


Here we have emitter reflected to base

$$I_E \Rightarrow I_B = \frac{I_E}{(\beta+1)}$$

$$R_E \Rightarrow R_E(\beta+1)$$

AC small signal Equivalent Circuit



1) $$A_V = \frac{v_o}{v_i}$$

$$A_V = \frac{v_o}{v_i} = \frac{v_o}{i_b} \cdot \frac{i_b}{v_i}$$

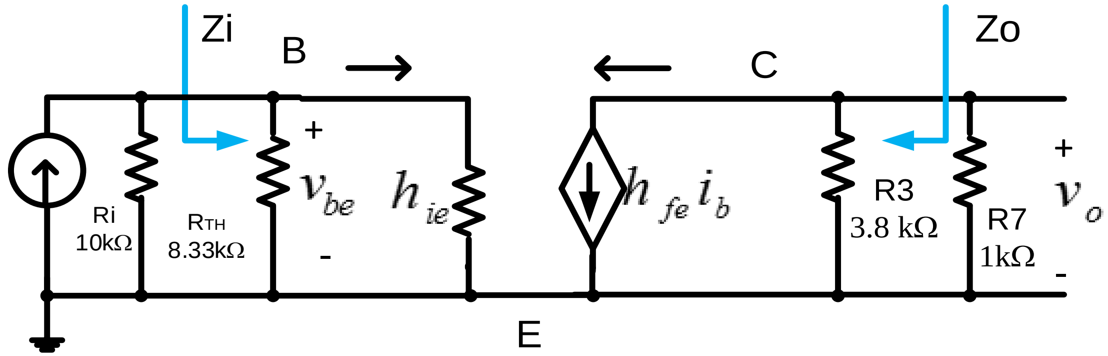
$$v_o = -h_{fe} i_b . (R_3 // R_7)$$

$$i_b = \frac{v_i}{h_{ie}}$$

$$= -h_{fe} . (R_3 // R_7) . \left(\frac{1}{h_{ie}} \right)$$

$$= -50 . (3.8\,k\Omega // 1\,k\Omega) . \left(\frac{1}{928\,\Omega} \right) = -42.7$$

AC small signal Equivalent Circuit



$$2)\ Z_I = R_{TH} // h_{ie}$$

$$= 8.33\ \text{k}\Omega // 928\ \Omega$$

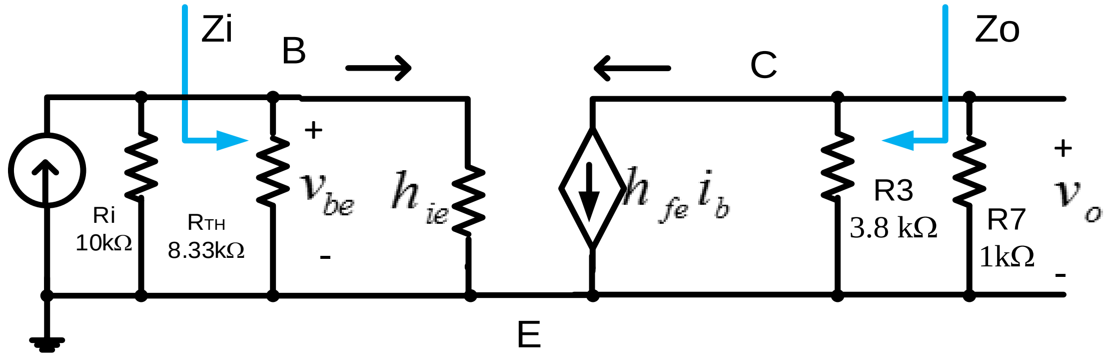
only elements to the right of arrow are considered

according to the given direction of the arrow

$$3)\ Z_o \Big|_{\text{all independant sources killed (i.e. Vi=0 or short)}} = 3.8\ \text{k}\Omega$$

$$\text{here } h_{fe}.i_b = 0 \ \text{ since } i_b = 0 \ \ (vi = 0 \ \ \text{- killed})$$

AC small signal Equivalent Circuit



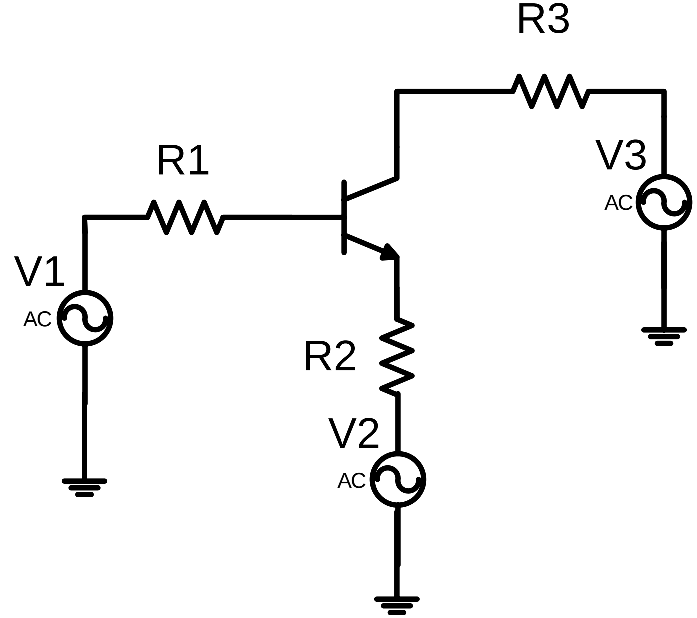
4) $A_i = \dfrac{i_o}{i_i}$

$$i_o = -h_{fe} i_b \left(\frac{R_3}{R_3 + R_7} \right)$$

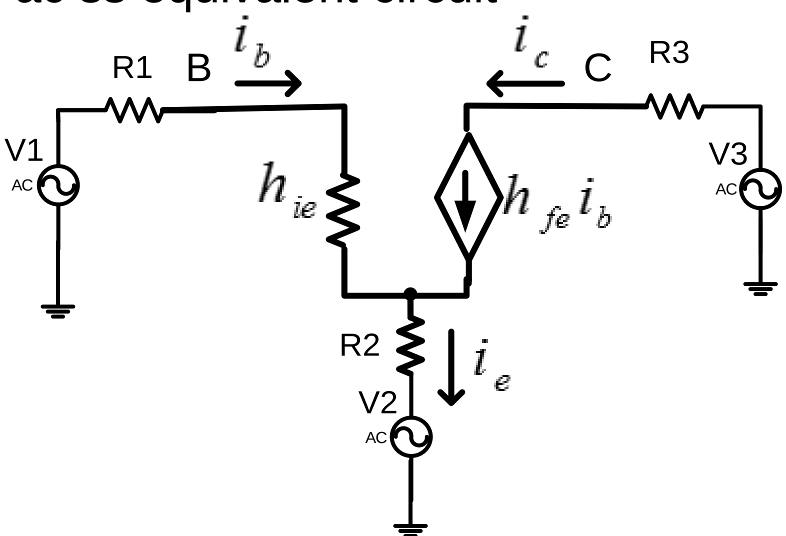
$$i_b = (i_i) \left(\frac{R_I // R_{TH}}{(R_I // R_{TH}) + h_{ie}} \right)$$

$$A_i = \frac{i_o}{i_i} = \frac{i_o}{i_b} \cdot \frac{i_b}{i_i} = -h_{fe} \left(\frac{R_3}{R_3 + R_7} \right) \cdot \left(\frac{R_I // R_{TH}}{(R_I // R_{TH}) + h_{ie}} \right) = -33$$

Impedance Reflection



ac ss equivalent circuit



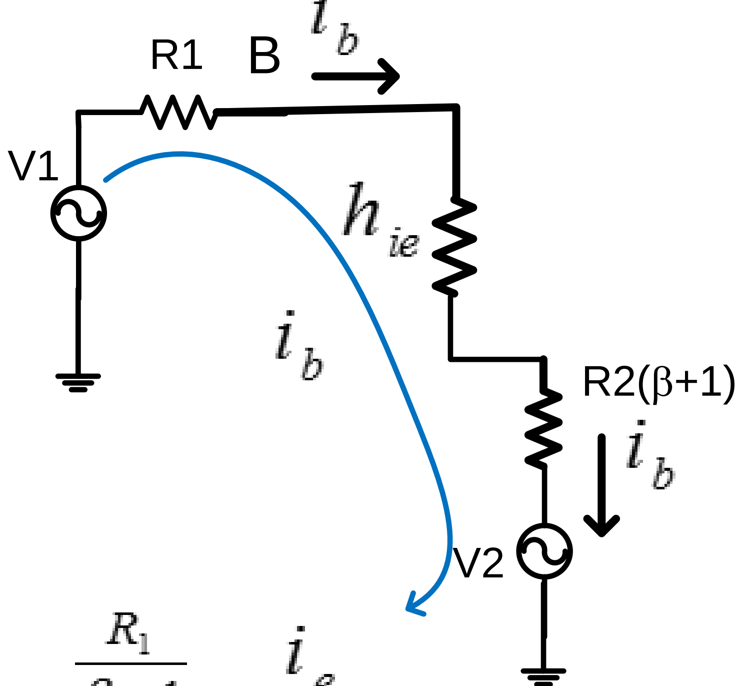
$$v_1 = R_1.i_b + h_{ie}.i_b + R_2.i_e + v_2$$

but $i_e = (\beta + 1)i_b$

$$v_1 = R_1.i_b + h_{ie}.i_b + R_2.(\beta + 1)i_b + v_2$$

$$i_b = \frac{v_1 - v_2}{R_1 + R_2.(\beta + 1)} \Leftarrow \text{base loop}$$

equivalent circuit equation



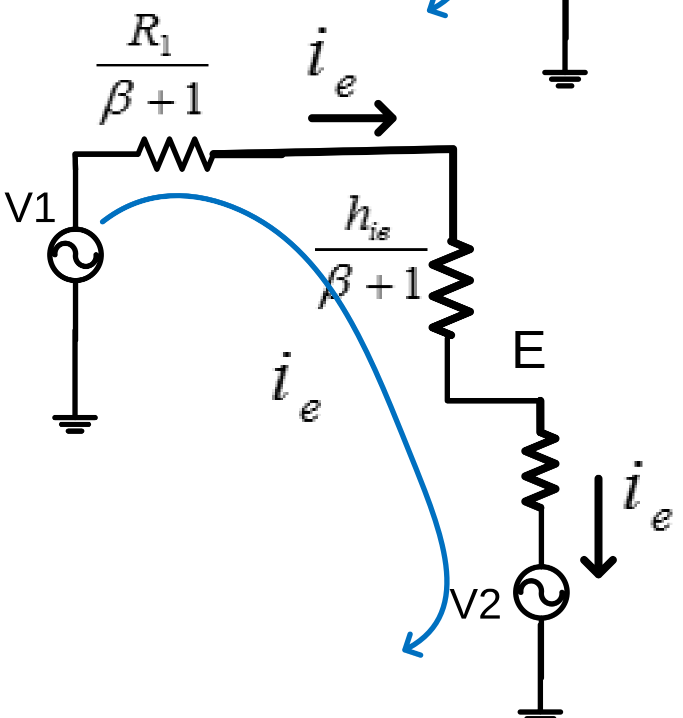
$$i_b = \frac{v_1 - v_2}{R_1 + h_{ie} + R_2.(\beta+1)} \Leftarrow \text{base loop}$$

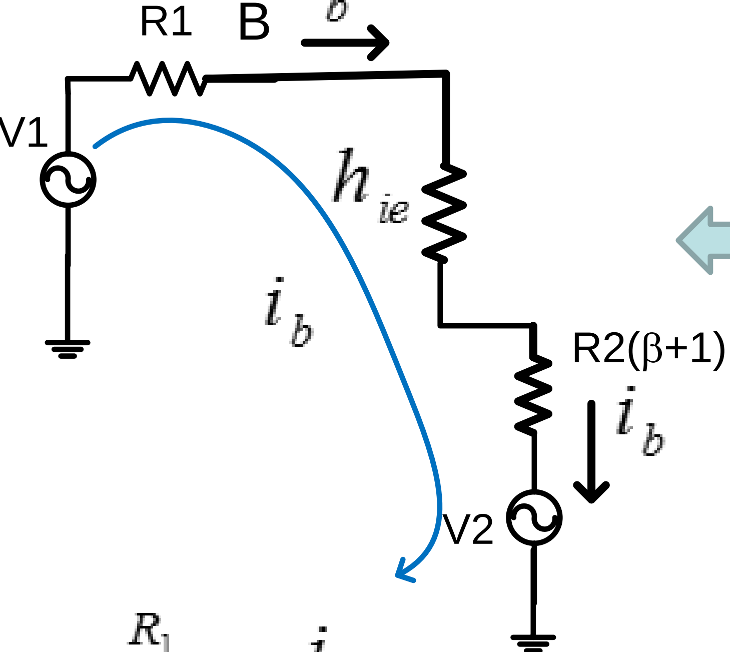
equivalent circuit equation

Instead of i_b use i_e

$$i_e = \frac{v_1 - v_2}{\frac{R_1}{(\beta+1)} + \frac{h_{ie}}{(\beta+1)} + R_2} \Leftarrow \text{Emitter loop}$$

equivalent circuit equation

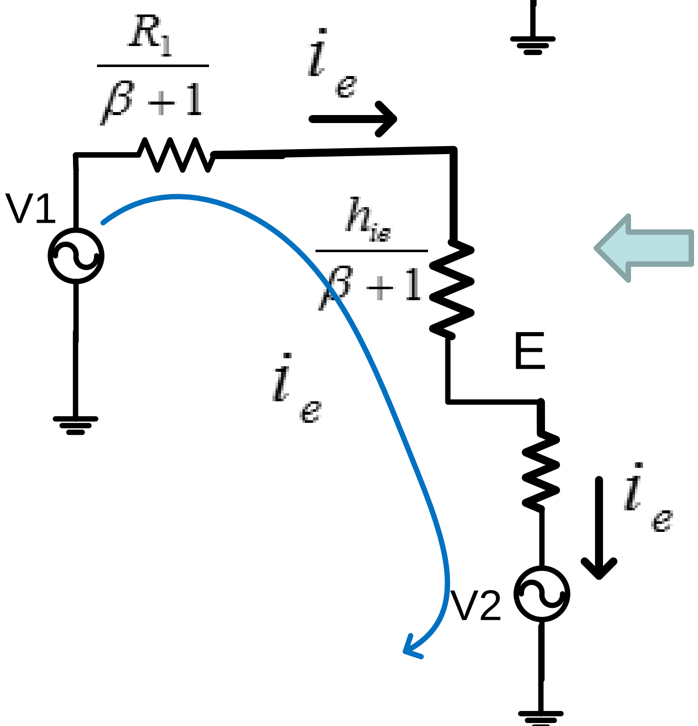




base equivalent circuit

(reflection from emitter to base)

Here we must change i_e to i_b which requires division by $(h_{fe}+1)$, but voltage must remain the same and thus the resistance must be multiplied by the same factor $(h_{fe}+1)$

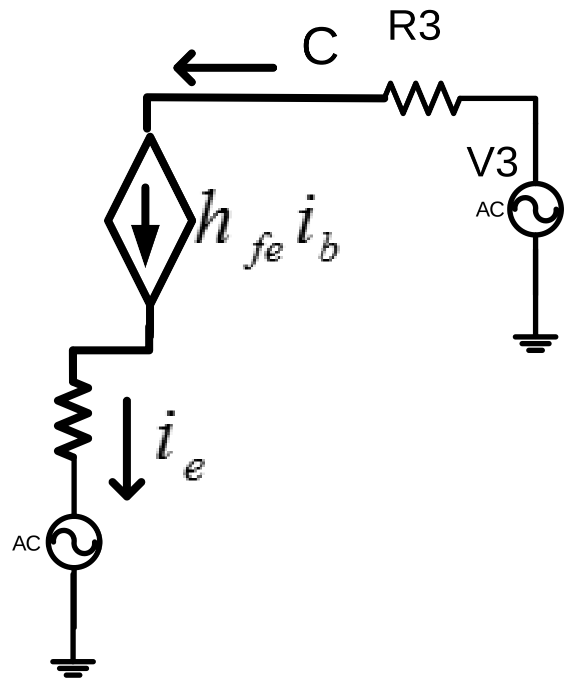


Emitter equivalent circuit

(reflection from base to emitter)

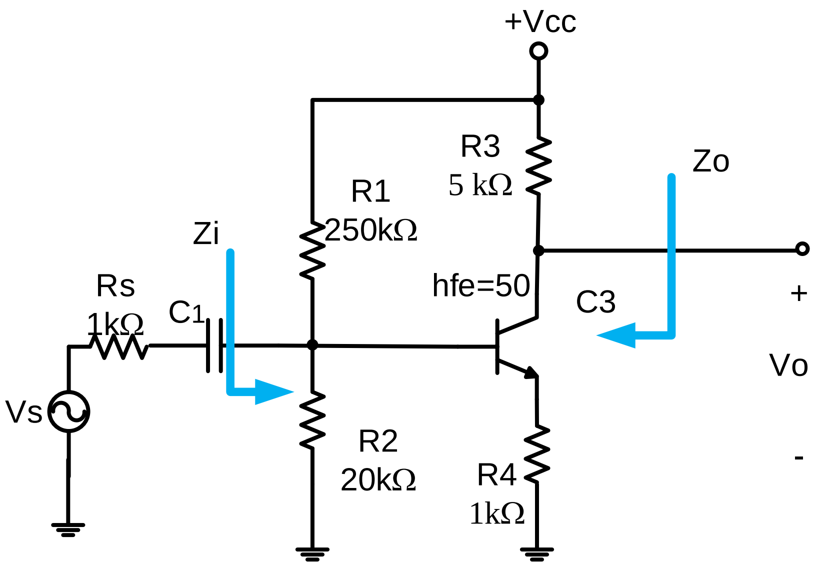
Here we must change i_b to i_e which requires multiplication by $(h_{fe}+1)$, but voltage must remain the same and thus the resistance must be divided by the same factor $(h_{fe}+1)$

Collector Equivalent Circuit



Note: there is no reflection from emitter to collector or vise vesra since the i_e and i_c are almost the same

Common Collector Amplifier



AC small signal Equivalent Circuit